Corresponding Parts of Congruent Figures Are Congruent

Common Core Math Standards

The student is expected to:

COMMON CORE **G-CO.B.7**

Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

Mathematical Practices

COMMON CORE **MP.2 Reasoning**

Language Objective

Have students fill in sentence stems to explain why figures are congruent or noncongruent.

ENGAGE

Essential Question: What can you conclude about two figures that are congruent?

The corresponding parts are congruent, and relationships within the figures, such as relative distances between vertices, are equal.

PREVIEW: LESSON PERFORMANCE TASK

View the online Engage. Discuss the photo and ask students to identify congruent shapes in the design. Then preview the Lesson Performance Task.

Name ______________________ Class ______________ Date ________

3.3 Corresponding Parts of Congruent Figures Are Congruent

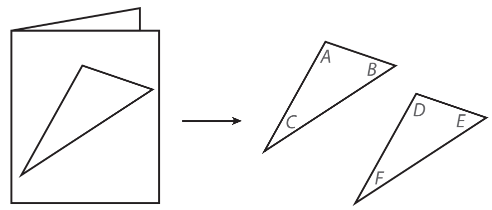
Resource Locker

Essential Question: What can you conclude about two figures that are congruent?

Explore Exploring Congruence of Parts of Transformed Figures

You will investigate some conclusions you can make when you know that two figures are congruent.

(A) Fold a sheet of paper in half. Use a straightedge to draw a triangle on the folded sheet. Then cut out the triangle, cutting through both layers of paper to produce two congruent triangles. Label them $\triangle ABC$ and $\triangle DEF$, as shown.



(B) Place the triangles next to each other on a desktop. Since the triangles are congruent, there must be a sequence of rigid motions that maps $\triangle ABC$ to $\triangle DEF$. Describe the sequence of rigid motions.

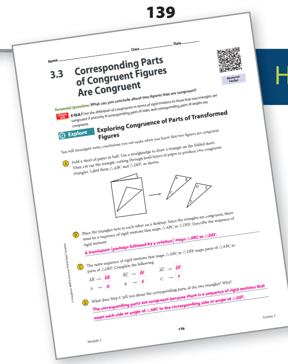
A translation (perhaps followed by a rotation) maps $\triangle ABC$ to $\triangle DEF$.

(C) The same sequence of rigid motions that maps $\triangle ABC$ to $\triangle DEF$ maps parts of $\triangle ABC$ to parts of $\triangle DEF$. Complete the following.

$\overline{AB} \rightarrow$ **$\overline{DE}$**	$\overline{BC} \rightarrow$ **$\overline{EF}$**	$\overline{AC} \rightarrow$ **$\overline{DF}$**
$A \rightarrow$ **D**	$B \rightarrow$ **E**	$C \rightarrow$ **F**

(D) What does Step C tell you about the corresponding parts of the two triangles? Why?

The corresponding parts are congruent because there is a sequence of rigid motions that maps each side or angle of $\triangle ABC$ to the corresponding side or angle of $\triangle DEF$.



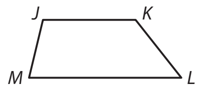
HARDCOVER PAGES 123–132

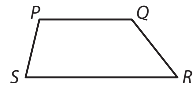
Turn to these pages to find this lesson in the hardcover student edition.

Reflect

1. If you know that $\triangle ABC \cong \triangle DEF$, what six congruence statements about segments and angles can you write? Why?
$\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, $\overline{AC} \cong \overline{DF}$, $\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$. The rigid motions that map $\triangle ABC$ to $\triangle DEF$ also map the sides and angles of $\triangle ABC$ to the corresponding sides and angles of $\triangle DEF$, which establishes congruence.

2. Do your findings in this Explore apply to figures other than triangles? For instance, if you know that quadrilaterals *JKLM* and *PQRS* are congruent, can you make any conclusions about corresponding parts? Why or why not?





Yes; since quadrilateral *JKLM* is congruent to quadrilateral *PQRS*, there is a sequence of rigid motions that maps *JKLM* to *PQRS*. This same sequence of rigid motions maps sides and angles of *JKLM* to the corresponding sides and angles of *PQRS*.

Explain 1 Corresponding Parts of Congruent Figures Are Congruent

The following true statement summarizes what you discovered in the Explore.

Corresponding Parts of Congruent Figures Are Congruent
If two figures are congruent, then corresponding sides are congruent and corresponding angles are congruent.

Example 1 $\triangle ABC \cong \triangle DEF$. **Find the given side length or angle measure.**

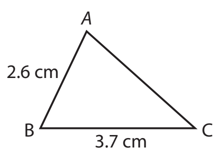
Ⓐ *DE*

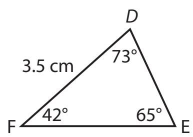
Step 1 Find the side that corresponds to $\overline{DE}$.

Since $\triangle ABC \cong \triangle DEF$, $\overline{AB} \cong \overline{DE}$.

Step 2 Find the unknown length.

$DE = AB$, and $AB = 2.6$ cm, so $DE = 2.6$ cm.





Ⓑ $m\angle B$

Step 1 Find the angle that corresponds to $\angle B$.

Since $\triangle ABC \cong \triangle DEF$, $\angle B \cong \angle$ **E**.

Step 2 Find the unknown angle measure.

$m\angle B = m\angle$ **E**, and $m\angle$ **E** $=$ **65**°, so $m\angle B =$ **65**°.

EXPLORE

Exploring Congruence of Parts of Transformed Figures

QUESTIONING STRATEGIES

When you are given two congruent triangles, how many pairs of corresponding parts—angles and sides—are there? **6; 3 angles and 3 sides**

EXPLAIN 1

Corresponding Parts of Congruent Figures Are Congruent

INTEGRATE MATHEMATICAL PRACTICES

Focus on Communication

MP.3 Have a student read the statement about Corresponding Parts of Congruent Figures. Discuss the meaning of the statement for general figures and then in terms of two triangles. Emphasize that the statement is a *biconditional*, an if-and-only-if statement that is true when read as an if-then statement in either direction.

PROFESSIONAL DEVELOPMENT

Math Background

In this lesson, students learn that if two figures (including triangles) are congruent, then corresponding pairs of sides and corresponding pairs of angles of the figures are congruent. This follows readily from the rigid-motion definition of congruence and from the statement that Corresponding Parts of Congruent Figures Are Congruent. This statement is a biconditional, a statement that is true in either direction. That is, if corresponding pairs of sides and corresponding pairs of angles in two figures are congruent, then the figures are congruent.

QUESTIONING STRATEGIES

? How do you determine which sides of two congruent figures correspond? **Use the order of letters in the congruence statement. The first letters correspond, the last letters correspond, and the other letters correspond in the same order.**

VISUAL CUES

Have each student make a poster illustrating the concept of congruent figures. The illustrations should be labeled to show which pairs of corresponding parts are congruent. Have them show both examples and non-examples of congruent figures in the poster.

EXPLAIN 2

Applying the Properties of Congruence

INTEGRATE MATHEMATICAL PRACTICES

Focus on Modeling

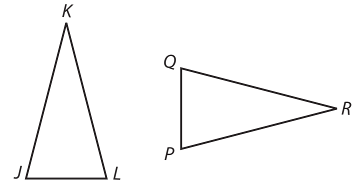
MP.4 Suggest that students list all the congruencies that relate the parts of the figures and mark the figures to show them. Once they have clearly represented the corresponding parts, they can more easily answer the questions.

QUESTIONING STRATEGIES

? How could you use transformations to decide whether two figures are congruent? **You could use transformations to create all pairs of corresponding parts congruent. Then the statement applies because if corresponding parts of congruent figures are congruent, then the figures are congruent.**

Reflect

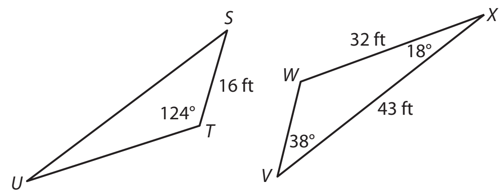
3. **Discussion** The triangles shown in the figure are congruent. Can you conclude that $\overline{JK} \cong \overline{QR}$? Explain.



No; the segments appear to be congruent, but the correspondence between the triangles is not given, so you cannot assume $\overline{JK}$ and $\overline{QR}$ are corresponding parts.

Your Turn

$\triangle STU \cong \triangle VWX$. Find the given side length or angle measure.



4. SU

Since $\triangle STU \cong \triangle VWX$, $\overline{SU} \cong \overline{VX}$.

$SU = VX = 43$ ft.

5. $m\angle S$

Since $\triangle STU \cong \triangle VWX$, $\angle S \cong \angle V$.

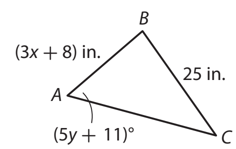
$m\angle S = m\angle V = 38°$.

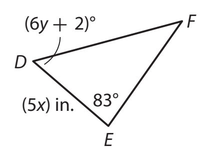
Explain 2 Applying the Properties of Congruence

Rigid motions preserve length and angle measure. This means that congruent segments have the same length, so $\overline{UV} \cong \overline{XY}$ implies $UV = XY$ and vice versa. In the same way, congruent angles have the same measure, so $\angle J \cong \angle K$ implies $m\angle J = m\angle K$ and vice versa.

Properties of Congruence	
Reflexive Property of Congruence	$\overline{AB} \cong \overline{AB}$
Symmetric Property of Congruence	If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AD}$.
Transitive Property of Congruence	If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

Example 2 **$\triangle ABC \cong \triangle DEF$. Find the given side length or angle measure.**





(A) AB

Since $\triangle ABC \cong \triangle DEF$, $\overline{AB} \cong \overline{DE}$.
Therefore, $AB = DE$.

Write an equation. $3x + 8 = 5x$

Subtract $3x$ from each side. $8 = 2x$

Divide each side by 2. $4 = x$

So, $AB = 3x + 8 = 3(4) + 8 = 12 + 8 = 20$ in.

COLLABORATIVE LEARNING

Small Group Activity

Have each student draw a pair of congruent figures on paper. Instruct them to switch papers and to write a congruence statement for the pair of figures. Then have them switch papers several more times within groups, write new congruence statements that fit the pair of figures, and list the congruent pairs of corresponding parts of the figures.

Ⓑ m∠D

Since $\triangle ABC \cong \triangle DEF$, $\angle$ **A** $\cong \angle D$. Therefore, m$\angle$ **A** $=$ m$\angle D$.

Write an equation. $5y +$ **11** $=$ **6y** $+ 2$

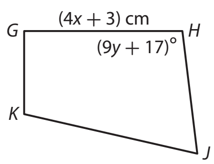
Subtract $5y$ from each side. $11 =$ **y** $+ 2$

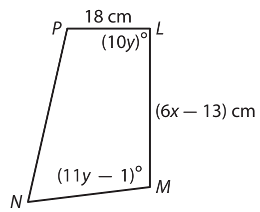
Subtract 2 from each side. **9** $=$ **y**

So, $m\angle D = (6y + 2)^\circ = \left(6 \cdot \boxed{9} + 2\right)^\circ = \boxed{56}^\circ$.

Your Turn

Quadrilateral $GHJK \cong$ quadrilateral $LMNP$. Find the given side length or angle measure.





6. LM

Since $GHJK \cong LMNP$, $\overline{GH} \cong \overline{LM}$. Therefore, $GH = LM$.

$4x + 3 = 6x - 13 \rightarrow 8 = x$

$LM = 6x - 13 = 6(8) - 13 = 35$ cm

7. m$\angle H$

Since quadrilateral $GHJK \cong$ quadrilateral $LMNP$, $\angle H \cong \angle M$. Therefore, $m\angle H = m\angle M$.

$9y + 17 = 11y - 1 \rightarrow 9 = y$

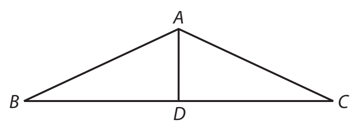
$m\angle H = (9y + 17)^\circ = (9 \cdot 9 + 17)^\circ = 98^\circ$

Explain 3 Using Congruent Corresponding Parts in a Proof

Example 3 Write each proof.

Ⓐ Given: $\triangle ABD \cong \triangle ACD$

Prove: D is the midpoint of $\overline{BC}$.



Statements	Reasons
1. $\triangle ABD \cong \triangle ACD$	1. Given
2. $\overline{BD} \cong \overline{CD}$	2. Corresponding parts of congruent figures are congruent.
3. D is the midpoint of $\overline{BC}$.	3. Definition of midpoint.

DIFFERENTIATE INSTRUCTION

Technology

Have students use geometry software to create designs using congruent triangles. They should arrange multiple congruent triangles using different colors, positions, and orientations. Ask them to make three separate designs: one using congruent equilateral triangles, one using congruent isosceles triangles, and one using congruent scalene triangles.

AVOID COMMON ERRORS

Students may correctly solve for a variable but then incorrectly give the value of the variable as a side length or angle measure. Remind them to examine the diagram carefully; sometimes a side length or angle measure is described by an expression containing a variable, not by the variable alone.

EXPLAIN 3

Using Congruent Corresponding Parts in a Proof

INTEGRATE MATHEMATICAL PRACTICES

Focus on Technology

MP.5 Encourage students to use geometry software to reflect the triangle with the given conditions and then to verify that corresponding congruent parts have equal measure.

CONNECT VOCABULARY EL

In this lesson, students learn the *Corresponding Parts of Congruent Figures Are Congruent*. Although acronyms (such as CPCTC) may be helpful to some students when referring to statements, postulates, or theorems, such devices may be a bit more difficult for English Learners at the Emerging level. Consider making a poster or having students create or copy a list of theorems, along with their meanings, for them to refer to in this module. Students may want to come up with a mnemonic for the CPCTC itself, such as Cooks Pick Carrots Too Carefully.

QUESTIONING STRATEGIES

Why do pairs of corresponding congruent parts have equal measure? **Since rigid motions preserve angle measure and length, and since there is a sequence of rigid motions that maps a figure to a congruent figure, pairs of corresponding parts must have equal measure.**

ELABORATE

INTEGRATE MATHEMATICAL PRACTICES

Focus on Modeling

MP.4 When examining congruent figures, students can see how each vertex is mapped to its corresponding vertex by designating corresponding vertices in the same color and using a different color for each pair of corresponding vertices. Students can also highlight pairs of corresponding sides in the same color, using a different color for each pair.

QUESTIONING STRATEGIES

Can you say two figures are congruent if their corresponding angles have the same measure? Explain. **No. You must also determine that the corresponding sides have the same measure.**

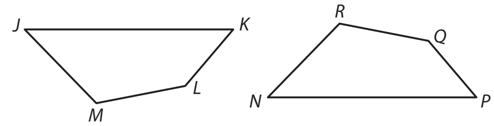
Can you say that a pair of corresponding sides of two congruent figures has equal measure? **Yes. If the figures are congruent, then each pair of corresponding sides is congruent and therefore has equal measure.**

SUMMARIZE THE LESSON

Suppose you know that $\triangle CBA \cong \triangle EFG$. What are six congruency statements? **$\angle C \cong \angle E$, $\angle B \cong \angle F$, $\angle A \cong \angle G$, $\overline{CB} \cong \overline{EF}$, $\overline{CA} \cong \overline{EG}$, $\overline{BA} \cong \overline{FG}$**

(B) Given: Quadrilateral $JKLM \cong$ quadrilateral $NPQR$; $\angle J \cong \angle K$

Prove: $\angle J \cong \angle P$



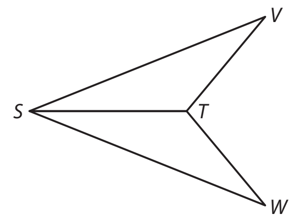
Statements	Reasons
1. Quadrilateral $JKLM \cong$ quadrilateral $NPQR$	1. **Given**
2. $\angle J \cong \angle K$	2. **Given**
3. $\angle K \cong \angle P$	3. **Corresponding parts of congruent figures are congruent.**
4. $\angle J \cong \angle P$	4. **Transitive Property of Congruence**

Your Turn

Write each proof.

8. Given: $\triangle SVT \cong \triangle SWT$

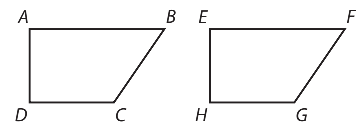
Prove: $\overline{ST}$ bisects $\angle VSW$.



Statements	Reasons
1. $\triangle SVT \cong \triangle SWT$	**1. Given**
2. $\angle VST \cong \angle WST$	**2. Corresponding parts of congruent figures are congruent.**
3. $\overline{ST}$ bisects $\angle VSW$.	**3. Definition of angle bisector.**

9. Given: Quadrilateral $ABCD \cong$ quadrilateral $EFGH$; $\overline{AD} \cong \overline{CD}$

Prove: $\overline{AD} \cong \overline{GH}$



Statements	Reasons
1. Quadrilateral $ABCD \cong$ quadrilateral $EFGH$	**1. Given**
2. $\overline{AD} \cong \overline{CD}$	**2. Given**
3. $\overline{CD} \cong \overline{GH}$	**3. Corresponding parts of congruent figures are congruent.**
4. $\overline{AD} \cong \overline{GH}$	**4. Transitive Property of Congruence**

LANGUAGE SUPPORT EL

Connect Vocabulary

Have students work in pairs. Provide each student with a protractor and ruler, and ask them to explain why two figures are congruent or noncongruent. Provide students with sentence stems to help them describe the attributes of the figures. For example: "The two (triangles/quadrilaterals/figures) are or are not congruent because their corresponding angles have/don't have equal measures. Angles ___ and ____ are corresponding, and measure _____ degrees. Corresponding sides have equal/not equal lengths." Students work together to complete the sentences.

Elaborate

10. A student claims that any two congruent triangles must have the same perimeter. Do you agree? Explain.

Yes; since the corresponding sides of congruent triangles are congruent, the sum of the lengths of the sides (perimeter) must be the same for both triangles.

11. If $\triangle PQR$ is a right triangle and $\triangle PQR \cong \triangle XYZ$, does $\triangle XYZ$ have to be a right triangle? Why or why not?

Yes; since $\triangle PQR$ is a right triangle, one of its angles is a right angle. Since corresponding parts of congruent figures are congruent, one of the angles of $\triangle XYZ$ must also be a right angle, which means $\triangle XYZ$ is a right triangle.

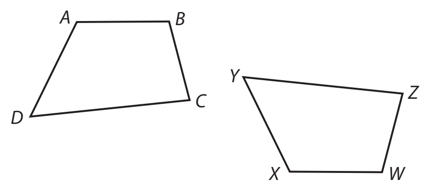
12. **Essential Question Check-In** Suppose you know that pentagon *ABCDE* is congruent to pentagon *FGHJK*. How many additional congruence statements can you write using corresponding parts of the pentagons? Explain.

There are five statements using the congruent corresponding sides and five statements using the congruent corresponding angles.

Evaluate: Homework and Practice

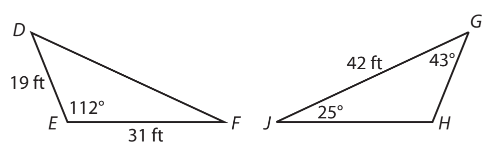
Personal Math Trainer
- Online Homework
- Hints and Help
- Extra Practice

1. Danielle finds that she can use a translation and a reflection to make quadrilateral *ABCD* fit perfectly on top of quadrilateral *WXYZ*. What congruence statements can Danielle write using the sides and angles of the quadrilaterals? Why?



The same sequence of rigid motions that maps *ABCD* to *WXYZ* also maps sides and angles of *ABCD* to corresponding sides and angles of *WXYZ*. Therefore, those sides and angles are congruent: $\overline{AB} \cong \overline{WX}$, $\overline{BC} \cong \overline{XY}$, $\overline{CD} \cong \overline{YZ}$, $\overline{AD} \cong \overline{WZ}$, $\angle A \cong \angle W$, $\angle B \cong \angle X$, $\angle C \cong \angle Y$, $\angle D \cong \angle Z$.

$\triangle DEF \cong \triangle GHJ$. Find the given side length or angle measure.



2. *JH*

Since $\triangle DEF \cong \triangle GHJ$, $\overline{FE} \cong \overline{JH}$. $FE = JH = 31$ ft, so $JH = 31$ ft.

3. $m\angle D$

Since $\triangle DEF \cong \triangle GHJ$, $\angle D \cong \angle G$. $m\angle D = m\angle G = 43°$

EVALUATE

Personal Math Trainer

ASSIGNMENT GUIDE

Concepts and Skills	Practice
Explore Exploring Congruence of Parts of Transformed Figures	Exercises 1
Example 1 Corresponding Parts of Congruent Figures are Congruent	Exercises 2–5, 10–13
Example 2 Applying the Properties of Congruence	Exercises 6–9
Example 3 Using Congruent Corresponding Parts in a Proof	Exercises 14–16

INTEGRATE MATHEMATICAL PRACTICES

Focus on Math Connections

MP.1 Have students consider whether two quadrilaterals, both with side lengths of 1 foot on each side, are congruent. Students should recognize that the description is that of a rhombus. Demonstrate that a box with an open top and bottom lying on its side is not rigid, and although the side lengths stay the same when one side is pushed, the angles change. Thus it is possible for the two figures described to have different angle measures and not be congruent.

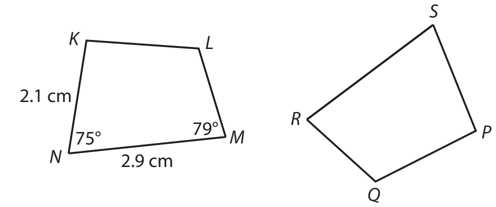
Exercise	Depth of Knowledge (D.O.K.)	COMMON CORE Mathematical Practices
1	**1** Recall of Information	**MP.6** Precision
2–5	**1** Recall of Information	**MP.2** Reasoning
6–9	**1** Recall of Information	**MP.4** Modeling
14–16	**2** Skills/Concepts	**MP.3** Logic
10–13, 17–18	**2** Skills/Concepts	**MP.2** Reasoning
19–22	**2** Skills/Concepts	**MP.4** Modeling

INTEGRATE MATHEMATICAL PRACTICES

Focus on Communication

MP.3 Have students compare their congruence statements for a given diagram, and ask them to write other correct congruence statements for the same diagram. Then have them write a congruence statement that is not correct for the diagram and explain why it is not correct.

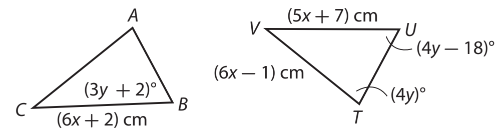
***KLMN* ≅ *PQRS*. Find the given side length or angle measure.**



4. $m\angle R$ **$\angle M \cong \angle R$. $m\angle M = m\angle R = 79°$.**

5. PS **$\overline{KN} \cong \overline{PS}$. $KN = PS =$ 2.1 cm**

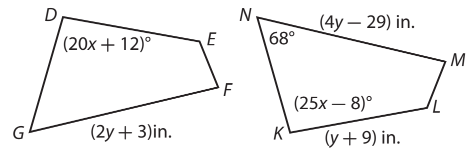
$\triangle ABC \cong \triangle TUV$. Find the given side length or angle measure.



6. BC **$\overline{BC} \cong \overline{UV}$. So, $BC = UV$.**
$6x + 2 = 5x + 7 \rightarrow x = 5$
So, $BC = 6x + 2 = 6(5) + 2 =$
$30 + 2 = 32$ cm.

7. $m\angle U$ **$\angle B \cong \angle U$. So, $m\angle B = m\angle U$.**
$3y + 2 = 4y - 18 \rightarrow 20 = y$
So, $m\angle U = (4y - 18)° =$
$(4 \cdot 20 - 18)° = 62°$.

***DEFG* ≅ *KLMN*. Find the given side length or angle measure.**



8. FG **$\overline{FG} \cong \overline{MN}$. So, $FG = MN$.**
$2y + 3 = 4y - 29 \rightarrow 16 = y$
So, $FG = 2y + 3 = 2(16) + 3 =$
$32 + 3 = 35$ in.

9. $m\angle D$ **$\angle D \cong \angle K$. So, $m\angle D = m\angle K$.**
$20x + 12 = 25x - 8 \rightarrow 4 = x$
So, $m\angle D = (20x + 12)° =$
$(20 \cdot 4 + 12)° = 92°$.

$\triangle GHJ \cong \triangle PQR$ and $\triangle PQR \cong \triangle STU$. Complete the following using a side or angle of $\triangle STU$. Justify your answers.

10. $\overline{GH} \cong$ **$\overline{ST}$**
$\triangle GHJ \cong \triangle STU$ by the Transitive Prop. of Cong., and corr. parts of ≅ fig. ≅.

11. $\angle J \cong$ **$\angle U$**
$\triangle GHJ \cong \triangle STU$ by the Transitive Prop. of Cong., and corr. parts of ≅ fig. ≅.

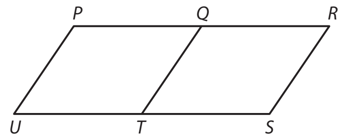
12. $GJ =$ **SU**
$\triangle GHJ \cong \triangle STU$ by the Transitive Prop. of Cong., and corr. parts of ≅ fig. ≅. Congruent segments have the same length.

13. $m\angle G =$ **$m\angle S$**
$\triangle GHJ \cong \triangle STU$ by the Transitive Prop. of Cong., and corr. parts of ≅ fig. ≅. Cong. angles have the same measure.

Exercise	Depth of Knowledge (D.O.K.)	COMMON CORE Mathematical Practices
23	**2** Skills/Concepts	**MP.2** Reasoning
24–25	**3** Strategic Thinking H.O.T.	**MP.3** Logic
26	**3** Strategic Thinking H.O.T.	**MP.6** Precision
27	**3** Strategic Thinking H.O.T.	**MP.3** Logic

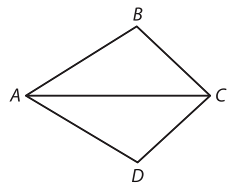
Write each proof.

14. Given: Quadrilateral $PQTU \cong$ quadrilateral $QRST$
Prove: $\overline{QT}$ bisects $\overline{PR}$.



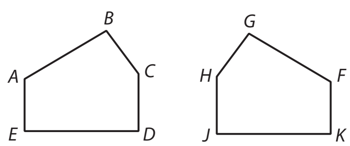
Statements	Reasons
1. Quadrilateral $PQTU \cong$ quadrilateral $QRST$	1. Given
2. $\overline{PQ} \cong \overline{QR}$	2. Corr. parts of $\cong$ fig. are $\cong$
3. Q is the midpoint of $\overline{PR}$.	3. Definition of midpoint
4. $\overline{QT}$ bisects $\overline{PR}$.	4. Definition of segment bisector

15. Given: $\triangle ABC \cong \triangle ADC$
Prove: $\overline{AC}$ bisects $\angle BAD$ and $\overline{AC}$ bisects $\angle BCD$.



Statements	Reasons
1. $\triangle ABC \cong \triangle DEF$	1. Given
2. $\angle BAC \cong \angle DAC$	2. Corr. parts of $\cong$ fig. are $\cong$
3. $\angle BCA \cong \angle DCA$	3. Corr. parts of $\cong$ fig. are $\cong$
4. $\overline{AC}$ bisects $\angle BAD$ and $\overline{AC}$ bisects $\angle BCD$.	4. Definition of angle bisector

16. Given: Pentagon $ABCDE \cong$ pentagon $FGHJK$; $\angle D \cong \angle E$
Prove: $\angle D \cong \angle K$



Statements	Reasons
1. Pentagon $ABCDE \cong$ pentagon $FGHJK$	1. Given
2. $\angle D \cong \angle E$	2. Given
3. $\angle E \cong \angle K$	3. Corr. parts of $\cong$ fig. are $\cong$
4. $\angle D \cong \angle K$	4. Transitive Property of Congruence

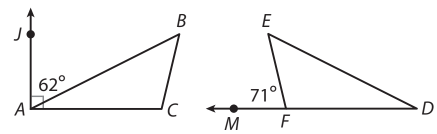
AVOID COMMON ERRORS

Students may find the value of a variable or the value of an algebraic expression as the solution to a problem when they are in fact only part of the way through the solving process. Remind students to always go back to the initial question to make sure the answer is the solution to the problem.

AVOID COMMON ERRORS

Students may write incorrect congruence statements. Make sure they understand that the order of the vertices in a congruence statement is not random. They should know that they can identify corresponding angles by choosing pairs of letters in corresponding positions in a congruence statement. For example, in $\triangle JZQ \cong \triangle MDH$, the letters J and M both appear in the first position in the names of their respective triangles. This means $\angle J \cong \angle M$. In a similar way, pairs of letters that are in corresponding positions yield pairs of corresponding sides.

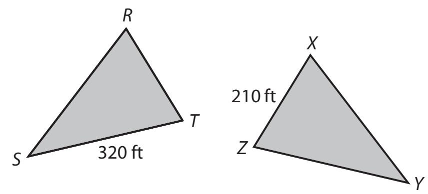
$\triangle ABC \cong \triangle DEF$. Find the given side length or angle measure.



17. m$\angle D$ **m$\angle JAB$ + m$\angle BAC$ = 90°, so 62° + m$\angle BAC$ = 90° and m$\angle BAC$ = 28°.**
Since $\triangle ABC \cong \triangle DEF$, $\angle BAC \cong \angle D$. m$\angle BAC$ = m$\angle D$, and m$\angle BAC$ = 28°, so m$\angle D$ = 28°.

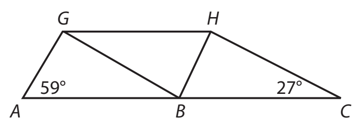
18. m$\angle C$ **m$\angle EFM$ + m$\angle EFD$ = 180°, so 71° + m$\angle EFD$ = 180° and m$\angle EFD$ = 109°.**
Since $\triangle ABC \cong \triangle DEF$, $\angle C \cong \angle EFD$. m$\angle C$ = m$\angle EFD$, and m$\angle EFD$ = 109°, so m$\angle C$ = 109°.

19. The figure shows the dimensions of two city parks, where $\triangle RST \cong \triangle XYZ$ and $\overline{YX} \cong \overline{YZ}$. A city employee wants to order new fences to surround both parks. What is the total length of the fences required to surround the parks?



Since $\triangle RST \cong \triangle XYZ$, $\overline{ST} \cong \overline{YZ}$, so $ST = YZ = 320$ ft. Since $\overline{YX} \cong \overline{YZ}$, $YX = YZ = 320$ ft.
Since the triangles are congruent, they have the same perimeter, which is $210 + 320 + 320 = 850$ ft. The total length of the fences is $850 + 850 = 1700$ ft.

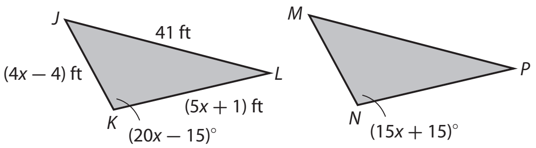
20. A tower crane is used to lift steel, concrete, and building materials at construction sites. The figure shows part of the horizontal beam of a tower crane, in which $\triangle ABG \cong \triangle BCH \cong \triangle HGB$



a. Is it possible to determine m$\angle GBH$? If so, how? If not, why not?
Yes; since corr. parts are $\cong$, m$\angle ABG$ = 27° and m$\angle HBC$ = 59°, so
m$\angle GBH$ = 180° − 59° − 27° = 94°.

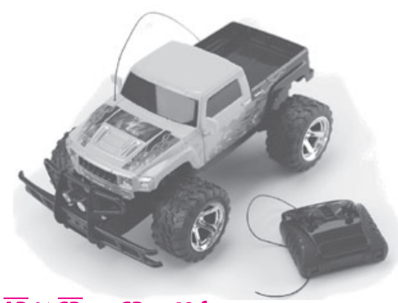
b. A member of the construction crew claims that $\overline{AC}$ is twice as long as $\overline{AB}$. Do you agree? Explain.
Yes; since corr. parts are $\cong$, $\overline{AB} \cong \overline{BC}$ and so B is the midpoint of $\overline{AC}$.
This means AC is twice AB.

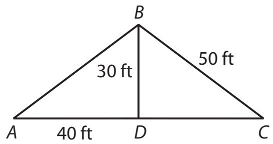
21. Multi-Step A company installs triangular pools at hotels. All of the pools are congruent and $\triangle JKL \cong \triangle MNP$ in the figure. What is the perimeter of each pool?



Since corresponding parts are congruent, $\angle K \cong \angle N$ and so $m\angle K = m\angle N$.
$20x - 15 = 15x + 15 \rightarrow x = 6$; $JK = 4(6) - 4 = 20$ ft, $KL = 5(6) + 1 = 31$ ft
The perimeter of $\triangle JKL$ is $20 + 31 + 41 = 92$ ft. The perimeter of $\triangle MNP$ is also 92 ft.

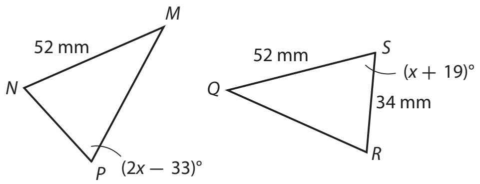
22. Kendall and Ava lay out the course shown below for their radio-controlled trucks. In the figure, $\triangle ABD \cong \triangle CBD$. The trucks travel at a constant speed of 15 feet per second. How long does it take a truck to travel on the course from A to B to C to D? Round to the nearest tenth of a second.





Since $\triangle ABD \cong \triangle CBD$, $\overline{AB} \cong \overline{CB}$, so $AB = 50$ ft. Also, $\overline{AD} \cong \overline{CD}$, so $CD = 40$ ft.
$AB + BC + CD = 50 + 50 + 40 = 140$ ft; distance = rate × time, so $140 = 15t \rightarrow t \approx 9.3$ s.

23. $\triangle MNP \cong \triangle QRS$. Determine whether each statement about the triangles is true or false. Select the correct answer for each lettered part.



a.	$\triangle QRS$ is isosceles.	● True	○ False
b.	$\overline{MP}$ is longer than $\overline{MN}$.	○ True	● False
c.	$m\angle P = 52°$	○ True	● False
d.	The perimeter of $\triangle QRS$ is 120 mm.	○ True	● False
e.	$\angle M \cong \angle Q$	● True	○ False

a. Since $\triangle MNP \cong \triangle QRS$, $\overline{MN} \cong \overline{QR}$, so $QR = 52$ mm. $\triangle QRS$ has two sides with the same length, so it is isosceles.

b. Since $\triangle MNP \cong \triangle QRS$, $\overline{MP} \cong \overline{QS}$, so $MP = 52$ mm. Therefore, $MP = MN$.

c. Since $\triangle MNP \cong \triangle QRS$, $\angle P \cong \angle S$, so $2x - 33 = x + 19$. Solving the equation shows that $x = 52$ and $m\angle P = (2x - 33)° = (2 \cdot 52 - 33)° = 71°$.

d. Since $\triangle MNP \cong \triangle QRS$, $\overline{MP} \cong \overline{QR}$, so $QR = 52$ mm. The perimeter of $\triangle QRS$ is $52 + 52 + 34 = 138$ mm.

e. Since $\triangle MNP \cong \triangle QRS$, $\angle M \cong \angle Q$ since corresponding parts of congruent figures are congruent.

INTEGRATE MATHEMATICAL PRACTICES

Focus on Reasoning

MP.2 When students solve algebraic equations to find the measures of congruent corresponding parts of figures, caution them to first verify that the correspondences are correct. Suggest that students start by listing the pairs of corresponding parts.

PEER-TO-PEER DISCUSSION

Ask students to discuss with a partner how to determine whether two figures are congruent. Have students give each other a pair of figures, look for the congruent corresponding parts, and then write a congruence statement for the figures. Repeat the exercise for other pairs of figures.

JOURNAL

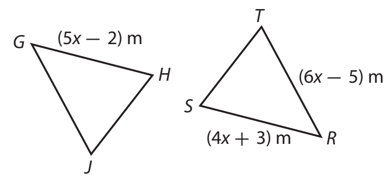
Have students write a journal entry in which they discuss the statement that Corresponding Parts of Congruent Figures Are Congruent in their own words. Encourage them to include one or more labeled figures as part of the journal entry.

H.O.T. Focus on Higher Order Thinking

24. Justify Reasoning Given that $\triangle ABC \cong \triangle DEF$, $AB = 2.7$ ft, and $AC = 3.4$ ft, is it possible to determine the length of $\overline{EF}$? If so, find the length and justify your steps. If not, explain why not.

No; the side of $\triangle ABC$ that corresponds to $\overline{EF}$ is $\overline{BC}$. The length of this side is not known and cannot be determined from the given information.

25. Explain the Error A student was told that $\triangle GHJ \cong \triangle RST$ and was asked to find GH. The student's work is shown below. Explain the error and find the correct answer.



Student's Work

$5x - 2 = 6x - 5$

$-2 = x - 5$

$3 = x$

$GH = 5x - 2 = 5(3) - 2 = 13$ m

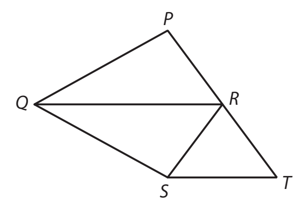
The student incorrectly identified corresponding sides. Since $\triangle GHJ \cong \triangle RST$, $\overline{GH} \cong \overline{RS}$.

$5x - 2 = 4x + 3 \rightarrow x = 5$; $GH = 5(5) - 2 = 23$ m.

26. Critical Thinking In $\triangle ABC$, $m\angle A = 55°$, $m\angle B = 50°$, and $m\angle C = 75°$. In $\triangle DEF$, $m\angle E = 50°$, and $m\angle F = 65°$. Is it possible for the triangles to be congruent? Explain.

No; if the triangles were congruent, then corresponding angles would be congruent. Since $m\angle F = 65°$, there is no angle of $\triangle ABC$ that could be the corresponding angle to $\angle F$, so the triangles cannot be congruent.

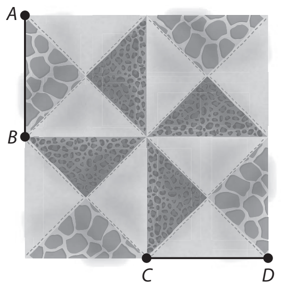
27. Analyze Relationships $\triangle PQR \cong \triangle SQR$ and $\overline{RS} \cong \overline{RT}$. A student said that point R appears to be the midpoint of $\overline{PT}$. Is it possible to prove this? If so, write the proof. If not, explain why not. **Yes;**



Statements	Reasons
1. $\triangle PQR \cong \triangle SQR$	1. Given
2. $\overline{RP} \cong \overline{RS}$	2. Corr parts of ≅ figs. are ≅
3. $\overline{RS} \cong \overline{RT}$	3. Given
4. $\overline{RP} \cong \overline{RT}$	4. Transitive Property
5. R is the midpoint of $\overline{PT}$	5. Definition of midpoint

Lesson Performance Task

The illustration shows a “Yankee Puzzle” quilt.



a. Use the idea of congruent shapes to describe the design of the quilt.

The design is created from 16 congruent triangles. Each quarter of the design consists of 4 of the triangles joined to form a square.

b. Explain how the triangle with base $\overline{AB}$ can be transformed to the position of the triangle with base $\overline{CD}$.

There are many ways to transform the triangle with base $\overline{AB}$ to the position of the triangle with base $\overline{CD}$. One way is to translate it to the position of the triangle directly beneath it, then, rotate it 90° counterclockwise about C, then translate to the right.

c. Explain how you know that $CD = AB$.

$CD = AB$ because corresponding parts of congruent figures are congruent.

EXTENSION ACTIVITY

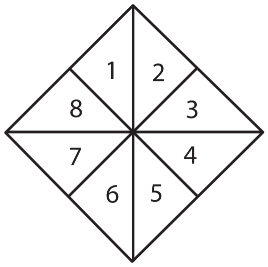
Challenge students to draw and color a design for a quilt that meets the following requirements:

- The design should be square.
- The design should consist of triangles and/or quadrilaterals only.
- The design should have 90-degree rotational symmetry.

INTEGRATE MATHEMATICAL PRACTICES

Focus on Patterns

MP.8 Sketch and number the eight inner triangles of the Yankee Puzzle quilt on the board.



Given Triangle 1, how could you find the locations of the other seven triangles using transformations? **Sample answer: Rotate Triangle 1 90°, 180°, and 270° clockwise around the center point to locate triangles 3, 5, and 7. Reflect Triangle 1 across the vertical center line to locate Triangle 2. Then rotate Triangle 2 90°, 180°, and 270° clockwise around the center point to locate triangles 4, 6, and 8.**

INTEGRATE MATHEMATICAL PRACTICES

Focus on Communication

MP.3 Describe how, starting with a square, you could draw the pattern of a Yankee Puzzle quilt. **Sample answer: Draw the diagonals of the square. Find the midpoints of the four sides. Connect the midpoint of each side with the midpoint of the side adjacent to it and the midpoint of the side opposite it.**

Scoring Rubric

2 points: Student correctly solves the problem and explains his/her reasoning.

1 point: Student shows good understanding of the problem but does not fully solve or explain his/her reasoning.

0 points: Student does not demonstrate understanding of the problem.